INTRODUCTION TO SHTUKAS AND HIGHER GROSS–ZAGIER FORMULA

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We study automorphic L-functions for GL(2) over a global field (e.g., those attached to elliptic curves). In the case of the rational number field, the Gross–Zagier formula expresses the first derivative in terms of the height of Heegner point on a modular curve. In the case of function fields, in a joint work [16, 17, 1] with Zhiwei Yun, we found a family of algebraic cycles on the moduli of rank two Drinfeld Shtukas, and we prove that their intersection numbers give higher order derivatives of L-function. The lectures will review the backgrounds and we will present the main ingredients in the proof. If time permits, we also discuss several constructions of algebraic cycles on Shimura varieties over number fields.

Some recommended prerequisites

(1) (moduli space of) vector Bundles over a curve, some reference [12, 13, 14].

(2) Basics on perverse sheaves. (Perverse sheaves appear only in the very end of the proof and we will keep the use in a minimal way. They could also be treated as a black-box, though they are very useful in many areas.) Some reference: [2, 3, 6, 7, 8, 9, 10, 11, 15]; A survey [5]. A readable sheaf theoretical treatment for topological IH: [3, V]

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