Abstract: A fundamental problem in 4-dimensional topology is the following geography question: "which simply connected topological 4-manifolds admit a smooth structure?" After the celebrated work of Kirby-Siebenmann, Freedman, and Donaldson, the last uncharted territory of this geography question is the "11/8-Conjecture''. This conjecture, proposed by Matsumoto, states that for any smooth spin 4-manifold, the ratio of its second-Betti number and signature is least 11/8.

Furuta proved the ''10/8+2''-Theorem by studying the existence of certain Pin(2)-equivariant stable maps between representation spheres. In this talk, we will present a complete solution to this problem by analyzing the Pin(2)-equivariant Mahowald invariants of powers of certain Euler classes in the RO(Pin(2))-graded equivariant stable homotopy groups of spheres. In particular, we improve Furuta's result into a ''10/8+4''-Theorem. Furthermore, we show that within the current existing framework, this is the limit. For the proof, we use the technique of cell-diagrams, known results on the stable homotopy groups of spheres, and the j-based Atiyah-Hirzebruch spectral sequence.

This is joint work with Michael Hopkins, Jianfeng Lin and XiaoLin Danny Shi.