

SLOPES OF MODULAR FORMS AND THE GHOST CONJECTURE

The goal of this series of talks is to explain the recent work [LTXZ1] and [LTXZ2] on slopes of modular forms and its various arithmetic applications.

Tentative Schedule (12 talks version)

Time	Speaker	Content
April 7 9:00–10:30	Liang Xiao	Talk 1: Introduction to ghost conjecture and its local version. Review the ghost conjecture raised by Bergdall and Pollack in [BP16 ⁺], [BP17 ⁺]. Explain the formulation of local ghost conjecture following [LTXZ1, § 2], and discuss the relation between the global and the local version.
April 7 10:50–12:20	Liang Xiao	Talk 2: Application of ghost conjectures Give an overview of the proof of ghost conjecture and introduce its applications following [LTXZ2, § 1].
April 7 14:00–15:30	Ruochuan Liu	Talk 3: Abstract p-adic modular forms and corank theorem Introduce the notions of abstract classical, overconvergent and p -adic forms and the power basis on spaces of abstract overconvergent forms. Explain the theta map and Atkin-Lehner involution under the abstract setup following [LTXZ1, § 3]. Prove the (general) corank theorem and explain the intuition behind the ghost multiplicities in the definition of ghost series, following [LTXZ2, § 3].
April 14 9:00–10:30	Bin Zhao	Talk 4: Basic properties of ghost series Give the dimension formulas for the space of abstract classical forms. Following [LTXZ1, § 4], explain the implications below of the dimension formulas: a) a formula on the degrees of the coefficients of ghost series; b) compatibility of the ghost series with theta maps, the Atkin–Lehner involutions and the p -stabilization process; c) ghost duality of ghost series; d) an estimate of ‘old form slopes’.
April 14 10:50–12:20	Bin Zhao	Talk 5: Further properties of the ghost Newton polygon. Following [LTXZ1, § 5], introduce the notion of ‘near Steinberg range’, and use it to give a criterion to detect vertices of the Newton polygon of ghost series;

April 14 14:00–15:30	Bin Zhao	Talk 6: Integrality of slopes of ghost series at classical weights. Show the integrality property of slopes of ghost series at classical weights.
April 21 9:00–10:00	Ruochuan Liu	Talk 7: Rigidity of power series of ghost series type Follow [LTXZ2, § 9] to prove the finiteness of irreducible components of the zero locus of a power series of ghost series type, and hint at the later application to finiteness of irreducible components of eigencurves.
April 21 10:10–11:10	Ruochuan Liu	Talk 8: Triangulline deformation space a la Breuil–Hellmann–Schraen Recall triangulline deformation space of Breuil–Hellmann–Schraen and discuss the crystalline slopes on it.
April 21 11:30–12:30	Ruochuan Liu	Talk 9: On Paškūnas modules. Give a brief introduction to Paškūnas modules and local Langlands correspondence for $GL_2(\mathbb{Q}_p)$.
April 21 14:00–15:00	Liang Xiao	Talk 10: Crystalline slopes on triangulline deformation spaces: I Translate local ghost theorem to results of crystalline slopes on triangulline deformation spaces following [LTXZ2, § 7].
April 21 15:10–16:10	Liang Xiao	Talk 11: Crystalline slopes on triangulline deformation spaces: II Follow [LTXZ2, § 8] to deduce the ghost conjecture of Bergdall–Pollack from the local ghost conjecture. Then explain some arithmetic consequences, e.g. Breuil–Buzzard–Emeerton conjecture, Gouvêa conjecture, Gouvêa–Mazur conjecture, etc.
April 21 16:30–17:30	Bin Zhao	Talk 12: Proof of local ghost conjecture I Follow [LTXZ2, § 4] to explain how to explain how Lagrange interpolation can reduce the proof of local ghost conjecture to an estimate on the determinant of minors of the U_p -operator on the power basis.
April 28 9:00–10:00	Liang Xiao	Talk 13: Proof of local ghost conjecture II Follow [LTXZ2, § 6] to explain how to inductively prove the needed estimate, especially the cofactor expansion argument that reduces to an estimate that will follow from inductive hypothesis.

April 28 10:10–11:10	Bin Zhao	Talk 14: Proof of local ghost conjecture III Discuss the improved halo estimate and modified Mahler basis; follow [LTXZ2, § 3].
April 28 11:30–12:30	Bin Zhao	Talk 15: Proof of local ghost conjecture IV Follow [LTXZ2, § 5] to conclude the proof of local ghost conjecture, where we explain how to use the improved halo estimates to complete the inductive proof.

REFERENCES

- [BP16⁺] J. Bergdall and R. Pollack, Slopes of modular forms and the ghost conjecture, [arXiv:1607.04658](#), *to appear in IMRN*.
- [BP17⁺] J. Bergdall and R. Pollack, Slopes of modular forms and the ghost conjecture II, [arXiv:1710.01572](#), *to appear in the Trans. of the Amer. Math. Soc.*
- [LTXZ1] R. Liu, N.X. Truong, L. Xiao and B. Zhao, A local analogue of the ghost conjecture of Bergdall-Pollack, [arXiv:2206.15372v2](#)
- [LTXZ2] R. Liu, N.X. Truong, L. Xiao and B. Zhao, Slopes of modular forms and geometry of eigencurves, [arXiv:2302.07697](#)