Derivation of the Vlasov equation from quantum many-body Fermionic systems with singular interaction

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Abstract

We consider the combined mean-field and semiclassical limit for a system of the N fermions interacting through singular potentials. We prove the uniformly in the Planck constant h propagation of quantum moments for the Hartree–Fock equation with singular pair interaction potential of the form $|x-y|^{-a}$, including the Coulomb interaction. Using these estimates, we obtain quantitative bounds on the distance between solutions of the many-body Schrödinger equation and solutions of the Hartree–Fock and the Vlasov equations in Schatten norms. For $a \in (0, \frac{1}{2})$, we obtain global-in-time results when $N^{-\frac{1}{2}} \ll h < CN^{-\frac{1}{3}}$. In particular, it leads to the derivation of the Vlasov equation with singular potentials. For $a \in (\frac{1}{2}, 1]$, our results hold only on a small time scale $t \sim h^{a-\frac{1}{2}}$, or with an N-dependent cutoff. The talk is based on our recent works in [1, 2, 3]. This is a joint work with Laurent Lafleche and Chiara Saffirio. The talk will be delivered in English and is meant for the general audience.

References

[1] J. Chong, L. Lafleche, and C. Saffirio. From Many-Body Quantum Dynamics to the Hartree–Fock and Vlasov Equations with Singular Potentials. *arXiv:2103.10946*, pages 1–74, Mar. 2021.

[2] J. Chong, L. Lafleche, and C. Saffirio. Global-in-time Semiclassical Regularity for the Hartree–Fock Equation. *Journal of Mathematical Physics*, 63(8):081904, 2022.

[3] J. Chong, L. Lafleche, and C. Saffirio. On the semiclassical regularity of thermal equilibria. *arXiv:2208.07911*, pages 1–14, Aug. 2022.