TITLES AND ABSTRACTS

Ulrich bundles: to be or not to be

Arnaud Beauville (*Université de Nice*)

After explaining what are Ulrich bundles and why they are interesting, I'll focus on the major problem of the subject: the existence of such bundles on any smooth projective variety. I'll explain how the Serre construction of rank 2 bundles give a positive answer for some surfaces and threefolds. Then I'll discuss a property of Ulrich bundles on surfaces, which might lead to an example of surface with no Ulrich bundle.

Sums of three squares and Noether-Lefschetz loci

Olivier Benoist (*CNRS, Université de Strasbourg*)

It is a theorem of Hilbert that a real polynomial in two variables that is non-negative is a sum of four squares of rational functions. Moreover, Cassels, Ellison and Pfister have shown that there exist such polynomials that are not sums of three squares of rational functions. In this talk, we will explain why the polynomials that may be written as sums of three squares are dense in the set of non-negative polynomials. The proof is Hodge-theoretic.

Compactified Jacobians from the algebro-geometric and combinatorial point of view

Lucia Caporaso (*Roma Tre University*)

We give an overview of the theory of Compactified Jacobians for nodal curves passing through some results of A. Beauville and arriving at recent progress about the combinatorial and tropical side of the theory.

Topological methods for moduli and deformation to hypersurface embedding

Fabrizio Catanese (*Universität Bayreuth*)

Topological methods in moduli theory contemplate for instance the question: are manifolds Y homeomorphic, or homotopically equivalent, to a given manifold X, deformation equivalent to X? An interesting class is the class defined in joint work with Ingrid Bauer: an Inoue type manifold is a free quotient $X = W/G$, where G is a finite group acting freely, and W is a hypersurface in a projective classifying space Z. The simplest example occurs when G is trivial, and Z is an Abelian variety. Already this example motivates the question: assume that we have a family W_t such that W_t maps to a projective manifold Z_t and for $t \neq 0$ we get a hypersurface embedding. Can we then describe the map of W_0 into Z_0 ?

In joint work with Yongnam Lee we solved the question under the natural assumption that the canonical bundle is ample: we get exactly the iterated univariate coverings of normal type, and we can explicitly describe these families.

Remarks on arithmetic ampleness

François Charles (*Université Paris-Sud*)

We will discuss a basic setup for ampleness of hermitian line bundles over arithmetic varieties (joint work with Bost). We will show how basic geometric results (cohomology vanishing, Nakai-Moishezon, Bertini) translate in this setting.

The integral Hodge conjecture for the total space of families of cubic threefolds

Jean-Louis Colliot-Thélène (*CNRS, Université Paris-Sud, Université Paris-Saclay*)

In joint work with A. Pirutka, we prove that the third unramified cohomology group of a smooth cubic threefold over the function field of a complex curve vanishes. For this, we combine a method of C. Voisin with Galois descent on the Chow group of codimension 2 cycles. As a corollary, we show that the integral Hodge conjecture holds for degree 4 classes on smooth projective fourfolds equipped with a fibration over a curve, the generic fibre of which is a smooth cubic threefold, with no restriction on the singularities of the special fibres. In the first part of the talk I shall review how one goes back and forth from questions on cycles of codimension 2 to questions on the third unramified cohomology group.

The Prym-Green conjecture

Gavril Farkas (*Humboldt Universität zu Berlin*)

By analogy with Green's Conjecture on syzygies of canonical curves, the Prym-Green conjecture predicts that the resolution of a general level p paracanonical curve of genus g is natural, that is, as "small" as the geometry of the curve allows. I will discuss a complete solution to this conjecture for odd genus, recently obtained in joint work with M. Kemeny.

Distinguished cycles on varieties whose motives are of abelian type

Lie Fu (*Université Claude Bernard Lyon 1*)

I will report a joint work with Charles Vial on the study the Chow rings of algebraic varieties whose Chow motives are of abelian type. Using the theory of symmetrically distinguished cycles on abelian varieties developed by Peter O'Sullivan, we give natural liftings of algebraic cycles modulo numerical equivalence to algebraic cycles modulo rational equivalence, which we call distinguished cycles. The central question that we will address is whether the diagonal, small diagonal and Chern classes are distinguished cycles. This condition in particular implies that distinguished cycles form a subalgebra containing all Chern classes and compatible with Poincaré duality. Roughly speaking, distinguished cycles provide a splitting of the cohomological part of the conjectural Bloch-Beilinson filtration on Chow rings of such varieties. The original motivation is to understand Beauville's splitting property for holomorphic symplectic varieties. The talk will be mainly about plenty of examples where this theory works.

Motivic aspects of K3 surfaces

Daniel Huybrechts (*Universität Bonn*)

I will explain in what sense isogenous K3 surfaces have equivalent derived categories. Combined with a recent observation that derived equivalent K3 surfaces have isomorphic motives, this proves that isogenous K3 surfaces have isomorphic Chow motives. As a byproduct, we prove the Hodge conjecture for squares of K3 surfaces with CM, a result originally due to Buskin.

Fano contact manifolds and contact prolongations

Jun-Muk Hwang (*Korea Institute for Advanced Studies*)

It has been conjectured that a Fano manifold with a contact structure is homogeneous. Beauville proved the conjecture under the additional assumptions that its automorphism group is reductive and the rational map given by the sections of the contact line bundle is generically finite over its image. His proof used the theory of nilpotent orbits of semisimple Lie algebras.

We present an approach to the conjecture using varieties of minimal rational tangents. We have been able to prove that a Fano contact manifold is homogeneous if its varieties of minimal rational tangents form a locally trivial fibration outside a set of codimension 2. This gives a new proof of Beauville's result.

Measures of irrationality of algebraic varieties

Robert Lazarsfeld (*Stony Brook University*)

I will discuss a circle of results and open problems centered around the question of measuring how irrational an algebraic variety can be. If time permits, I will explain how this relates to a simple new proof of Rans theorem on secant lines. This is joint work with Bastianelli, DePoi, Ein and Ullery.

Supersingular irreducible symplectic varieties

Zhiyuan Li (*Shanghai Center for Mathematical Sciences*)

The supersingular K3 surfaces are first introduced by Artin via the formal Brauer group. Artin has conjectured that all supersingular K3 surface are unirational. Recently, this conjecture has been confirmed by Liedtke when $p > 3$. In this talk, we introduce various notions of supersingular irreducible symplectic varieties over a field of characteristic p, which are conjecturally equivalent. Moreover, there is a natural generalization of Artin's conjecture for irreducible symplectic varieties. We will show that this conjecture holds for most known examples of irreducible symplectic varieties. This is a joint work with Lie Fu.

Genus zero orbifolds and the Wiman-Edge pencil

Eduard Looijenga (*Yau Mathematical Sciences Center*)

The Riemann-Hurwitz formula shows that a genus 6 curve endowed with a faithful action of the icosahedral group has as orbit space a genus zero curve with four orbifold points. The question we address is: to what extend determines the orbifold its A_5-cover? The answer leads us to reconstruct the Wiman-Edge pencil and involves a cofinite subgroup of the modular group which is not a congruence subgroup. This represents joint work with Benson Farb and partly also with Igor Dolgachev.

On the recognition of geometric substructures on uniruled projective subvarieties

Ngaiming Mok (*The University of Hong Kong*)

The theory of varieties of minimal rational tangents (VMRTs) of Hwang-Mok is a differential-geometric theory on tangents to minimal rational curves on uniruled projective manifolds of Picard number 1. Associated to $(X, \mathcal{K})$ is the fibered space $\pi : \mathcal{C}(X) \to X$ of VMRTs, which we will call the VMRT structure on $(X, \mathcal{K})$. More recently, with Jaehyun Hong and Yunxin Zhang we have embarked on the study of germs of complex submanifolds S on uniruled projective manifolds inheriting geometric substructures obtained from intersections of VMRTs with tangent subspaces, giving rise to sub-VMRT structures $\varpi : \mathcal{C}(S) \to S$, $\mathcal{C}(S) := \mathcal{C}(X) \cap \mathbb{P}T(S)$. Central to the study of VMRT and sub-VMRT structures are various types of recognition problems, i.e., problems of characterizing special types of Fano manifolds of Picard number 1 or special uniruled projective subvarieties on them in terms of VMRTs and sub-VMRTs. When X is an irreducible Hermitian symmetric space of the compact type of rank ≥ 2 and S is modeled on a Schubert cycle $Z \subset X$, the recognition of $Z \subset X$ in terms of geometric substructures is related to the question of Schur rigidity, which was settled under the assumption that $Z \subset X$ is nonsingular by Walters and Bryant in special cases and by Hong (2007) in the general case using differential systems and methods of Lie algebra cohomology. We focused on the case where $Z \subset X$ is nonsingular, and proved (Mok-Zhang 2016) a result of rigidity of the pair (Z, X) consisting more generally of a rational homogeneous space $X = G/P$ of Picard number 1 corresponding to a marked Dynkin diagram and a rational homogeneous submanifold $Z \subset X$ obtained from a marked Dynkin subdiagram, in a general form applicable to uniruled projective manifolds, by introducing a notion of nondegeneracy of substructures analogous to but different from the notion of nondegeneracy for mappings introduced by Hong-Mok (2010). We introduce a quantitative measure for the degree of nondegeneracy for substructures, leading thereby to results on the recognition problem in special cases such as the characterization of maximal projective subspaces on smooth linear sections of sufficiently small codimension in G/P. For a uniruled projective manifold X we give sufficient conditions for $S \subset X$ to be saturated by lines and for S to be an open subset of a projective subvariety $Z \subset X$.

Coble surfaces and their automorphism groups

Shigeru Mukai (*Research Institute for Mathematical Sciences, Kyoto*)

A Coble surface is the blow-up of the projective plane at the singular points of a rational sextic curve which is allowed to be reducible. One can define its twisted Picard lattice and root system using the K3 double cover, similarly to Enriques surfaces. The automorphism group of a Coble surface is a discrete subgroup of the 2-dimensional Cremona group. It shrinks from a very infinite group to a mildly infinite group or a finite group under specialization, just like the Mordell-Weil group of a rational elliptic surface. After reviewing basic material, I will present some examples of automorphism groups, together with a system of generating Cremona transformations, and propose a problem on their virtual cohomological dimension.

OG10 and Prym varieties

Giulia Saccà (*Stony Brook University*)

I will start by recalling some recent results on O'Grady's 10 dimensional example of hyper-Kahler manifold (OG10), obtained in work with R. Laza and C.Voisin. One of the ingredients of these results is a theory of relative Prym varieties. I will focus on this aspect, showing how it can be used to adapt Beauville's method for counting rational curves on K3 surfaces to obtain a very quick calculation of the Euler characteristic of OG10 (part of joint work in progress with K. Hulek and R. Laza).

A finite dimensional proof of Verlinde formula

Xiaotao Sun (*AMSS, Chinese Academy of Sciences*)

By degenerating a smooth curve to a curve with one node (irreducible or reducible), we establish two recurrence relations for the dimensions of spaces of generalized theta functions on moduli spaces of semi-stable parabolic bundles on smooth curves of genus g, which imply an explicit formula of dimension (Verlinde formula).

There are two steps to establish such recurrence relations: (1) factorizations of generalized theta functions over nodal curves; (2) invariance of dimensions during degeneration, which are implied by vanishing theorem of cohomolgy on moduli spaces. The step (1) and step (2) for $g > 2$ were done by myself around 2000. However vanishing theorem for $g < 3$ remains open.

Recently, with one of my students, we prove that moduli spaces of semi-stable parabolic bundles and generalized parabolic sheaves with fixed determinants are of globally Frobenius regular type, which imply the vanishing theorem for any genus.

$K3$ categories and 0-cycles on holomorphic symplectic varieties

Qizheng Yin (*Beijing International Center for Mathematical Research*)

In joint work with Junliang Shen and Xiaolei Zhao, we studied the connections between the derived category of a K3 surface, 0-cycles on the K3 surface, and 0-cycles on the moduli spaces of stable objects in the derived category. Results include the proof of a conjecture of O'Grady (which builds upon previous work of Huybrechts, O'Grady, and Voisin) and some applications to the Beauville-Voisin conjecture for holomorphic symplectic varieties that arise as moduli spaces. More recently, we experimented a possible generalization of the picture to the Kuznetsov category of a cubic 4-fold. This is joint work in progress with Junliang Shen.