WORKSHOP ON IHARA'S LEMMA

The cohomology groups of modular curves play an important role in the study of many number theory problems, e.g., the modularity conjecture, mod p and p-adic Langlands correspondence, Iwasawa theory etc. In this workshop, we study the analogous properties for Shimura curves and some of the applications. We focus on one important property, known as *Ihara's lemma*, and study a recent preprint *Ihara's lemma for Shimura curves over totally real fields via patching* by Jeffrey Manning and Jack Shotton.

Let F be a totally real field. Let l be a prime number unramified in F. Let D be a quaternion algebra over F ramified at all but one infinite place. Let $K \subset (D \otimes \mathbb{A}_F^\infty)^\times$ be a compact open subgroup. I.e. we are in the standard setting for Shimura curves.

Let $\mathfrak{p}$ be a finite place of F at which K and D are unramified. We have a natural map

$$\pi^* : H^1(X_K, \mathbb{F}_l)^{\oplus 2} \to H^1(X_{K_0(\mathfrak{p})}, \mathbb{F}_l).$$

The main result of [1] is the following.

Theorem 0.1. *Under some technical conditions, the kernel of π^* is Eisenstein.*

The main idea of the proof is to transfer the problem into Ihara's Lemma for Shimura sets associated to definite quaternion algebras, which is known and not difficult to prove. In order to make the transfer, the authors construct a filtration of $H^1(X_{K_0(\mathfrak{p})}, \mathbb{F}_l)$ given by the geometry of integral models of the curves. The novelty of their proof is then to do a *patching* with respect to the filtration and obtain extra information.

The workshop consists of essentially five talks, each of which is about two hours long.

(1) Background and basic properties of Shimura curves. [1, Section 2].
(2) Explain the ultrapatching construction and explain the global deformation problem. [1, Section 4.1, 4.2].
(3) Apply the patching for Shimura curves and Shimura sets. [1, Sections 4.3, 4.4].
(4) Compute certain local deformation rings. [1, Section 3].
(5) Prove the main result. [1, Section 6].

References

[1] J. Manning, J. Shotton; *Ihara's lemma for Shimura curves over totally real fields via patching.* `https://arxiv.org/abs/1907.06043v1`

Thursday, August 29

9h30-10h30 Chuangxun Cheng
Title: Introduction to Ihara's lemma

10h45-11h45 Chuangxun Cheng
Title: Some properties of Shimura curves

14h-15h30 Liang Xiao
Title: Patching – general construction

15h45-17h15 Liang Xiao
Title: Patching – the case of Shimura curves and Shimura sets

Friday, August 30

9h30-10h30 Aditya Karnataki
Title: Local deformation rings at $\ell \neq p$

10h45-11h45 Aditya Karnataki
Title: Explicit computation of local deformation rings

14h00-16h00 Yongquan Hu
Title: Proof of Main theorem