Titles and Abstracts

1. Karin Baur (Universität Graz)

Orbifold diagrams and skew group categories

Alternating strand diagrams (as introduced by Postnikov) on the disk have been used in the study of the coordinate ring of the Grassmannian. In particular, they give rise to clusters of the Grassmannian cluster algebras (Scott) or to cluster-tilting objects of the Grassmannian cluster categories of Jensen-King-Su (Baur-King-Marsh).

On the other hand, orbifolds have also been related to cluster structures as Paquette-Schiffler (or Chekhov-Shapiro for a geometric approach). Here we introduce orbifold diagrams as quotients of symmetric Postnikov diagrams and show how to associate quivers with potentials to them. This is joint work with Andrea Pasquali (Stuttgart) and Diego Velasco (Cali).

2. Peigen Cao (Université de Paris)

Bongartz completion via c-vectors.

We first give a characterization for Bongartz completion in tau-tilting theory using c-vectors. Motivated by this characterization, we introduce Bongartz completion in cluster algebras. Then we give some properties and applications of Bongartz completion in cluster theory. This is a joint work with Y. Gyoda and T. Yurikusa.

3. Man-Wai Cheung (Harvard University)

Cluster varieties: compactification and holomorphic correspondence

Cluster varieties are log Calabi-Yau varieties which are unions of algebraic tori glued by birational "mutation" maps. They are blown up of the toric varieties. In toric geometry, projective toric varieties can be described by polytopes. We will see how to generalize the polytope construction to cluster convexity which satisfies piecewise linear structure. As an application, we will see that the non-integral vertex in the Newton Okounkov body of Grassmannian comes from broken line convexity. We will then move forward to discuss family Floer mirror construction and disk potential of the Lagrangian torus fibers via holomorphic/tropical correspondence for cluster varieties of rank 2. The talk will be based on a series of joint works with Bardwell-Evans, Bossinger, Hong, Lin, Magee, Najera-Chavez.

4. Ben Davison (University of Edinburg)

Strong positivity for quantum cluster algebras

Quantum cluster algebras are quantizations of cluster algebras, which are a class of algebras interpolating between integrable systems and combinatorics. These algebras were originally introduced to study positivity phenomena arising in the study of quantum groups, and so one of

the key questions regarding them (and their quantum analogues) is whether they admit a basis for which the structure constants are positive. The classical version of this question was settled in the affirmative by Gross, Hacking, Keel and Kontsevich. I will present a proof of the quantum version of this positivity for skew-symmetric quantum cluster algebras, due to joint work with Travis Mandel, based on results in categorified Donaldson-Thomas theory and using scattering diagrams.

5. Nanqing Ding (Nanjing University)

Recollements associated to cotorsion pairs over upper triangular matrix rings

Let A, B be two rings and $T = \begin{pmatrix} A & M \\ 0 & B \end{pmatrix}$ with M an A-B-bimodule. Suppose that we are given two complete hereditary cotorsion pairs $(\mathcal{A}_A, \mathcal{B}_A)$ and $(\mathcal{C}_B, \mathcal{D}_B)$ in A-Mod and B-Mod respectively. We define two cotorsion pairs $(\Phi(\mathcal{A}_A, \mathcal{C}_B), \mathrm{Rep}(\mathcal{B}_A, \mathcal{D}_B))$ and $(\mathrm{Rep}(\mathcal{A}_A, \mathcal{C}_B), \Psi(\mathcal{B}_A, \mathcal{D}_B))$ in T-Mod and show that both of these cotorsion pairs are complete and hereditary. If we are given two cofibrantly generated model structures $\mathcal{M}_A$ and $\mathcal{M}_B$ on A-Mod and B-Mod respectively, then using the result above, we investigate when there exists a cofibrantly generated model structure $\mathcal{M}_T$ on T-Mod and a recollement of $\mathrm{Ho}(\mathcal{M}_T)$ relative to $\mathrm{Ho}(\mathcal{M}_A)$ and $\mathrm{Ho}(\mathcal{M}_B)$. Finally, some applications are given in Gorenstein homological algebra. This talk is a report on joint work with R.M. Zhu and Y.Y. Peng.

6. Ming Fang (Academy of Mathematics and Systems Science of CAS)

Rigidity dimension of algebras

A new homological dimension, called rigidity dimension, is introduced to measure the quality of resolutions of finite dimensional algebras (especially of infinite global dimension) by algebras of finite global dimension and big dominant dimension. Upper bounds of the dimension are established in terms of extensions and of Hochschild cohomology, and finite-ness in general is derived from homological conjectures. In particular, the rigidity dimension of a non-semisimple group algebra is finite and bounded by the order of the group. Then invariance under stable equivalences is shown to hold, with some exceptions when there are nodes in case of additive equivalences, and without exceptions in case of triangulated equivalences. Stable equivalences of Morita type and derived equivalences, both between self-injective algebras, are shown to preserve rigidity dimension as well. Some examples are discussed.

7. Jiarui Fei (Shanghai Jiao Tong University)

Tropical F-polynomials and Cluster Algebras

The representation-theoretic interpretations of g-vectors and F-polynomials are two fundamental ingredients in the (additive) categorification of cluster algebras. We knew that the g-vectors are related to the presentation spaces. We introduce the tropical F-polynomial f_M of a quiver

representation M, and explain its interplay with the general presentation for any finite-dimensional basic algebra. As a consequence, we give a presentation of the Newton polytope N(M) of M. We propose an algorithm to determine the generic Newton polytopes, and show it works for path algebras. As an application, we give a representation-theoretic interpretation of Fock-Goncharov's cluster duality pairing. We also study many combinatorial aspects of N(M), such as faces, the dual fan and 1-skeleton. We conjecture that the coefficients of a cluster monomial corresponding to vertices are all 1, and the coefficients inside the Newton polytope are saturated. We show the conjecture holds for acyclic cluster algebras.

8. Daniel Labardini-Fragoso (National Autonomous University of Mexico)

Laminations, tau-reduced components, generic bases and bangle bases

In relatively recent joint work, Geiss, Schröer and I showed that there is a bijective correspondence between the set of laminations of an unpunctured surface (\Sigma, M) and the set of generical lytau-reduced irreducible components (formerly called strongly reduced irreducible components by Geiss-Leclerc-Schröer) of the representation spaces of the gentle Jacobian algebra of any triangulation of (\Sigma, M). We also showed that the generic basis of the corresponding (upper) cluster algebra coincides with Musiker-Schiffler-Williams' bangle basis in the unpunctured case. In this talk I will present these results and report on ongoing joint work with Geiss and Wilson that intends to extend them to the case of punctured cases.

9. Changjian Fu (Sichuan University)

c-vectors and f-vectors revisited

c-vectors and f-vectors are integer vectors arising from cluster theory, where c-vectors parameterize coefficients, while f-vectors are maximal degree vectors of F-polynomials. In this talk, we will review certain (new) properties and questions related to c- and f-vectors. Partially based on joint work with Y. Gyoda and with S. Geng and P. Liu.

10. Alexander Goncharov (Yale University)

Cluster structures on moduli spaces of non-commutative local systems on decorated surfaces

A decorated surface is a surface with punctures and a collection of special points on the boundary, modulo isotopies. We consider the moduli space of local systems of n-dimensional vector bundles over an arbitrary non-commutative field, aka skew field, R. When R is a commutative field, there is a collection of coordinate systems on this moduli space, equivariant under the action of the mapping class group of the surface, constructed by Vladimir Fock and myself.

I will explain what is the analog of this construction when R is an arbitrary skew field. In the case when n=2 this is closely related to (but yet different than) the work of A. Berenstain and V. Retakh on non-commutative cluster algebras related to surfaces. This is a joint work with Maxim Kontsevich.

11. Song He (Institute of Theoretical Physics of CAS)

Cluster algebras for scattering amplitudes

Cluster algebras have been known to play an important role in the study of scattering amplitudes in N=4 super-Yang-Mills via its Grassmannian geometric structures. In this talk I will review some recent progress in uncovering cluster-algebra structures for scattering of generalized particles and strings in a wider context. We propose a construction of generalized associahedra for finite-type cluster algebra, which nicely encode tree and one-loop amplitudes of a scalar theory; by considering vast generalizations of string amplitudes dubbed stringy canonical forms, we then define "cluster string integrals" and “cluster configuration spaces” for any Dynkin diagram, which for type A reduce to usual string amplitudes and moduli spaces (mainly based on 1912.08707, 1912.11764, 2005.11419).

12. Min Huang (Sun Yat-sen University)

Positivity for quantum cluster algebras from orbifolds

We report our recent progress on the positivity for the quantum cluster algebras from orbifolds.

13. Kiyoshi Igusa (Brandeis University)

Duality of signed exceptional sequences

There are (n+1) ^{n-1} exceptional sequences of type A_n. This number also counts the number of rooted forests with n numbered vertices. Olivier Bernardi and I formulated a conjecture relating doubly signed exceptional sequences of type A_n and a certain two-variable generating function for rooted forests. A key feature of the conjecture is that it treats projective and injective objects symmetrically. But it does not always hold.

This talk will review known facts about signed exceptional sequences, a concept invented by Gordana Todorov and myself, an extension to tau-tilting theory by Aslak-Bakke Buan and Bethany Marsh. Then I will attempt to make the theory symmetrical to try to understand when the conjecture might hold.

14. Osamu Iyama (University of Tokyo)

Wall-chamber structure in Grothendieck group and canonical decomposition

In representation theory of finite dimensional algebras A, torsion classes are an important class of subcategories of mod A, and correspond bijectively with intermediate t-structures in the derived category. The most basic class of torsion classes (called functorially finite) is parametrized by silting complexes. There are two natural generalizations of this class. One is called semistable torsion classes, and parametrized by elements of the real Grothendieck group K0(proj A) R of the category proj A of finitely generated projective A-modules. The other is called morphism torsion

classes, and parametrized by morphisms in the category proj A. We show that torsion classes in the first class can be described completely by those in the second class. We also discuss `TF equivalence relation' on K0 (proj A) R defined by semistable torsion classes. We show that the open cone given by Derksen-Fei's canonical decomposition is contained in a TF equivalence class. Moreover, they coincide if A is E-tame or hereditary. This talk is based on a joint work with Sota Asai.

15. Bernhard Keller (Université de Paris)

On mutation at frozen vertices

Mutations at frozen vertices were first used by Chris Fraser and Melissa Sherman-Bennett in their recent work on cluster structures on open positroid varieties. We will explore such mutations in the combinatorial, the algebraic and the categorical setting and show how they naturally appear in the study of Fraser's extended affine braid group action on the Grassmannian. This is a report on results from Yilin Wu's ongoing Ph. D. thesis.

16. Myungho Kim (Korea Institute for Advanced Study)

Cluster algebras and finite-dimensional modules over quantum affine algebras

In their pioneering work, Hernandez and Leclerc showed that the category C of finite-dimensional modules over quantum affine algebras has a rich structure closely related to the theory of cluster algebras. Based on this observation, they provided the notion of monoidal categorification of cluster algebras. In this talk, I will explain that many monoidal subcategories of C provide monoidal categorification of cluster algebras, containing the case conjectured by Hernandez and Leclerc. This talk is based on the joint works with Masaki Kashiwara, Se-jin Oh, and Euiyong Park.

17. Kyungyong Lee (University of Nebraska-Linocln)

Greedy bases revisited, with a view toward the Jacobian conjecture

The Jacobian conjecture concerns the automorphisms of polynomial rings. We present an approach to the two-dimensional Jacobian conjecture, which uses certain joint divisibility and support conditions. These conditions have been inspired by the theory of cluster algebras, in particular, the construction of Lee-Li-Zelevinsky's greedy basis for rank 2 cluster algebras. No prior knowledge on the Jacobian conjecture or greedy basis is necessary. This is joint work with Ethan Hurst, Li Li, and George Nasr.

18. Li Li (Oakland University)

Support of elements in cluster algebras

Newton polytopes provide a combinatorial approach to study Laurent polynomials through the convex hull of their supports. In general, it could be interesting to study a certain region that encloses the support of a Laurent polynomial, where such a region may not be convex, or not even a polytope. In this talk I will explain how this idea is used to study greedy bases, triangular bases and other special elements in cluster algebras.

19. Travis Mandel (University of Utah)

Bracelet bases are theta bases

Cluster algebras from marked surfaces have well-known collections of special elements called "bracelet bases", as do the corresponding quantizations and the associated (quantum) cluster Poisson algebras. I will review these constructions and explain upcoming joint work with Fan Qin in which we prove several conjectures regarding the (quantum) bracelet bases by proving that they coincide with the corresponding (quantum) theta bases.

20. Hiraku Nakajima (University of Tokyo)

Representations of shifted Yangian

Shifted Yangian and shifted quantized loop algebras for symmetrizable Kac-Moody Lie algebras are realized by quantized Coulomb branches of quiver gauge theories with symmetrizers. Their representations, including finite-dimensional ones, category O, and even more general ones, are understood by Lusztig's canonical bases of unfolded symmetric quantized Kac-Moody algebras and their framed versions (joint work with Weekes, arXiv:1907.06552). I will explain several consequences of this theory in examples.

21. Tomoki Nakanishi (Nagoya University)

Cluster patterns and scattering diagrams

We review the construction of scattering diagrams for cluster patterns by Gross, Hacking, Keel, and Kontsevich. Then, we show how these scattering diagrams are related with cluster patterns via C- and G-matrices and F-polynomials by a different approach from GHKK without relying on their toric/birational geometrical setting. The sign-coherence and Laurent positivity follow from this relation.

22. Pierre-Guy Plamondon (Université de Paris Sud XI)

On g-vector fans of cluster algebras

We will discuss properties of the g-vector fan of a cluster algebra obtained from the representation theory of algebras. In finite type, we classify all relations between g-vectors and describe the space f realizations of the generalized associahedra, whose normal fans are the-vector fans. This is done by looking at relations in the Grothendieck group of a sub-extriangulated structure of the cluster categories in finite types. We then discuss the cases where the g-vector fan is densen the real space that contains it by studying "truncated twists functors" on the derived categories of tame algebras.

23. Matthew Pressland (Universität Stuttgart)

Categorification of positroid varieties

A connected Postnikov diagram D, consisting of a set of crossing strands in the disc, determines both a cluster algebra structure on a corresponding positroid variety in the Grassmannian, and a (Frobenius) categorification of this cluster algebra. I will explain this construction, and its relationship to Jensen, King and Su's Grassmannian cluster category. A key tool is provided by perfect matchings of the dimer model associated to D, which correspond to certain modules for the endomorphism algebra of a cluster-tilting object in the categorification. If time permits, I will also explain how considering perfect matching modules allows us to relate Marsh and Scott's dimer partition function for twisted Plücker coordinates to the cluster character. This is partly joint work with İ. Çanakçı and A. King.

24. Yannan Qiu (Zhejiang University)

The based rings of two-sided cells in an affine Weyl group of type \tilde B_3

We introduce Lusztig's conjecture on the structure of the based ring of two-sided cell. And for type \tilde B_3 we show that Lusztig's conjecture on the structure of the based ring of two-sided cell corresponding to the unipotent class in Sp_6(\mathbb C) with 3 equal Jordan blocks needs modified. This is a joint work with Nanhua Xi.

25. Yu Qiu (Tsinghua University)

Calabi-Yau-X and cluster-X categories from decorated marked surfaces(=DMS)

We introduce the graded DMS to study the Calabi-Yau-X categories associated to double graded quivers with super potential from surfaces. In particular, this provides a link between Calabi-Yau-3 categories of quiver with potential from triangulated surfaces and topological Fukaya categories from surfaces. This is a joint work with Akishi Ikeda and Yu Zhou.

26. Shiquan Ruan (Xiamen University)

Tilting objects on tubular weighted projective lines: a cluster tilting approach

This is joint work with Jianmin Chen, Pin Liu and Yanan Lin. Using cluster tilting theory, we investigate tilting objects in the stable category of vector bundles on a weighted projective line of weight type (2, 2, 2, 2). More precisely, a tilting object consisting of rank-two bundles is constructed via cluster tilting mutation. Moreover, the cluster tilting approach also provides a new method to classify the endomorphism algebras of tilting objects in the category of coherent sheaves and the associated bounded derived category.

27. Ralf Schiffler (University of Connecticut)

On a geometric model for syzygies over 2-Calabi-Yau tilted algebras

In this talk, we report on joint work with Khrystyna Serhiyenko available at https://arxiv.org/abs/2106.06496. We consider the class of 2-Calabi-Yau tilted algebras that are defined by a quiver with potential whose dual graph is a tree. These algebras are wild in general. For every such algebra B, we construct a polygon S with a checkerboard pattern in its interior that gives rise to a category Diag(S). The indecomposable objects of Diag(S) are the 2-diagonals in S, and its morphisms are given by certain pivoting moves between the 2-diagonals. We conjecture that the category Diag(S) is equivalent to the stable syzygy category over the algebra B, such that the rotation of the polygon corresponds to the shift functor on the syzygies. In particular, the number of indecomposable syzygies is finite and the projective resolutions are periodic. We prove the conjecture in the special case where every chordless cycle in the quiver is of length three.

As a consequence, we obtain an explicit description of the projective resolutions. Moreover, we show that the syzygy category is equivalent to the 2-cluster category of type A, and we introduce a new derived invariant for the algebra B that can be read off easily from the quiver.

28. Peng Shan (Tsinghua University)

Cohomology of affine Springer fibers and center of small quantum groups

I will report some recent progress on relationship between cohomology of affine Springer fibers and center of small quantum groups. This is based on joint work with R. Bezrukavnikov, P. Boixeda-Alvarez and E. Vasserot.

29. Michael Shapiro (Michigan State University)

Noncommutative cluster integrability

We define a discrete dynamical system (non-commutative pentagram map) and prove its noncommutative integrability. To prove integrability we define non-commutative double quasi-Poisson bracket on the space of non-commutative arc weights of a directed graph on a

cylinder which gives rise to the quasi-Poisson bracket of Massuyeau and Turaev on the group algebra of the fundamental group of a surface. We show that the induced double quasi Poisson bracketon the boundary measurements can be described via non-commutative r-matrix formalism which gives a conceptual proof of the result by N. Ovenhouse that the traces of powers of Lax matrix form an infinity system of Hamiltonians in involution. (joint w/ N. Ovenhouse and S. Arthamonov)

30. Xiuping Su (University of Bath)

Auslander algebras and cluster algebras C[G/P] in type A

The Auslander algebra A of the truncated polynomial ring k[x]/(x^n) is quasi-hereditary. In this talk I will construct some subcategories of Delta-filtered A-modules (aka good A-modules) and define an exact structure on these subcategories. I will then show that these subcategories under the exact structure are stably 2-CY and explain how these categories lead to categorification of cluster algebras defined on partial flag varieties G/P of type A. I will also show that these subcategories provide natural quantum seeds and discuss relations between the q-cluster algebras constructed from the quantum seeds and quantum flag varieties. This talk is based on ongoing joint work with B T Jensen.

31. Gordana Todorov (Northeastern University College of Science)

Semi-invariant pictures of tame hereditary algebras near their null roots

We study local structure of semi-invariant pictures of tame hereditary algebras near their null roots. We define a construction that we call co-amalgamation, and show that this local structure is completely described by the semi-invariant pictures of a collection of Nakayama algebras.

32. Daping Weng (Michigan State University)

Augmentations, Fillings, and Clusters of Positive Braid Closures

In the study of Legendrian links, one important task is to distinguish different exact Lagrangian fillings of a Legendrian link, up to Hamiltonian isotopy, in symplectic R^4. We introduce a cluster K2 structure on the augmentation variety of the Chekanov-Eliashbergdga for the rainbow closure of any positive braid. Using the Eckholm-Honda-Kalman functor from the cobordism category of Legendrian links to the category of dga's, we prove that a big family of fillings give rise to cluster seeds on the augmentation variety, and these cluster seeds can be used to distinguish non-Hamiltonian isotopic fillings. Moreover, by relating a cyclic rotation concordance with the cluster Donaldson-Thomas transformation on the augmentation variety, we prove that, except for a family of positive braids that are associated with finite type quivers, the rainbow closure of all other positive braids admits infinitely many non-Hamiltonian isotopic fillings. This is joint work with H. Gao and L. Shen.

33. Quanshui Wu (Fudan University)

A generalization of Koszul algebra and Koszul duality

In the talk, we will first recall the notion of classical graded Koszul rings defined by Beilinson-Ginzburg-Soergel and ungraded Koszul rings defined by Martinéz-Villa and Green. Then we will give a definition of (weakly) Koszulity for $\mathbb{N}$-graded rings A such that A_0 is noetherian semiperfect, which unifies the classical graded case and the ungraded semiperfect case. Under some natural assumptions, we show that the following statements are equivalent: A is Koszul; the Yoneda Ext ring $E(A) \coloneqq \underline{Ext}^{\cdot}_A(A/J, A/J)$ of A is Koszul where J is the graded Jacobson radical of A; the associated ring $\mathrm{Gr}_J A$ of the radical filtration of A is classical Koszul; the opposite ring of A is Koszul. If A is Koszul, we prove that the double Yoneda Ext ring of A is isomorphic to $\mathrm{Gr}_J A$ as graded rings. This result degenerates to the classical Koszul duality when A is classical Koszul. It is also proved that when A is Koszul of finite global dimension then A is generalized Artin-Schelter regular if and only if the Yoneda ring $E(A)$ of A is self-injective.

This is a joint work with Haonan Li.

34. Jie Xiao (Tsinghua University)

The multiplication formulas of quantum cluster algebras (I)

This talk is based on joint work with Zhimin Chen and Fan Xu. In this talk, we first review the cluster characters for 2-Calabi-Yau triangulated categories and categories of nilpotent modules over preprojective algebras. By applying the property of Ext-symmetry and introducing weight function, we obtain the multiplication formulas for quantum cluster characters associated to abelian categories with the property of Ext-symmetry and 2-Calabi-Yau triangulated categories with tilting objects.

35. Fan Xu (Tsinghua University)

The multiplication formulas of quantum cluster algebras (II)

This talk is based on joint work with Zhimin Chen and Jie Xiao and a continuation of "The multiplication formulas of quantum cluster algebras (I)". In this talk, we generalize the Geiss-Leclerc-Schroeer isomorphism between certain varieties of partial composition series and the corresponding quiver Grassmannians and then make a connection between two multiplication formulas in the previous talk. We explain that the proper choice of weight function induces the mutation formula for quantum cluster characters given by Qin.

36. Yu Zhou (Tsinghua University)

A geometric model for perfect categories of graded skew-gentle algebras

Let A be a graded skew-gentle algebra, i.e., a Z-graded algebra which is finite-dimensional and skew-gentle when forgetting the grading. We classify indecomposable objects in the perfect category of A in terms of admissible curves with local systems on a graded punctured marked surface. For good objects, we describe the morphisms between them via tagged intersections. The cones of certain morphisms are also given. This is based on an ongoing joint work with Yu Qiu and Chao Zhang.