

# **2023 Number Theory, Dynamical Systems and Related Topics**

May 8-12, 2023

Morningside Center of Mathematics, CAS

## **Sponsors:**

Academy of Mathematics and Systems Science, CAS

Morningside Center of Mathematics, CAS

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The conference “2023 Number Theory, Dynamical Systems and Related Topics” will be held at **Morningside Center of Mathematics**, which was founded by Chinese Academy of Sciences in 1996 and **Professor Shing-Tung Yau** has been serving as the director of the center.

**Registration Date & Location:**

**May 8<sup>th</sup>, 2023, 8:30-9:00, MCM Building 110.**

Address: No. 55, Zhongguancun East Road, Haidian District, Beijing

地址：北京市海淀区中关村东路 55 号

**Conference Time: May 8-12, 2023**

**Conference Venue: MCM Building 110**

Address: No. 55, Zhongguancun East Road, Haidian District, Beijing

**Website:** [http://www.mcm.ac.cn/events/programs/202304/t20230424\\_740603.html](http://www.mcm.ac.cn/events/programs/202304/t20230424_740603.html)

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**WeChat QR code:**



Location:



## Invited Speakers

Shilei Fan	Central China Normal University
Hui Gao	Southern University of Science and Technology
Weikun He	Academy of Mathematics and Systems Science, CAS
Zhuchao Ji	Westlake University
Yao Li	Peking University
Chunhui Liu	Harbin Institute of Technology
Zheng Xiao	Peking University
Junyi Xie	Peking University
She Yang	Peking University
Hexi Ye	Zhejiang University
Shucheng Yu	University of Science and Technology of China
Runlin Zhang	Chongqing University
Peng Zheng	Université Paris Cité

## Organizers

Shaoshi Chen	Academy of Mathematics and Systems Science, CAS
Yongquan Hu	Morningside Center of Mathematics, CAS
Zhizhong Huang	Academy of Mathematics and Systems Science, CAS

# Conference Schedule

May 8 <sup>th</sup> , 2023 (Monday)		
9:00-10:00	Hexi Ye	A uniform bound for the number of common preperiodic points
10:00-10:30	Coffee Break	
10:30-11:30	Shucheng Yu	The light cone Siegel transform, its moment formulas and applications
11:30-13:30	Lunch Break	
13:30-14:30	Junyi Xie	DAO for curves I (MCM 210)
14:40-15:40	Junyi Xie	DAO for curves II (MCM 210)
15:40-16:00	Coffee Break	
16:00-17:00	Zhuchao Ji	Local rigidity of Julia sets I (MCM 210)
17:30	Banquet	
May 9 <sup>th</sup> , 2023 (Tuesday)		
9:00-10:00	Shilei Fan	Bohr chaoticity of dynamical systems
10:00-10:30	Coffee Break	
10:30-11:30	Junyi Xie	DAO for curves III
11:30-13:30	Lunch Break	
13:30-14:30	Zhuchao Ji	Local rigidity of Julia sets II
14:40-15:40	Zhuchao Ji	Local rigidity of Julia sets III
15:40-16:00	Coffee Break	
16:00-17:00	Weikun He	Spectral gap in some finite groups

May 10 <sup>th</sup> , 2023 (Wednesday)		
9:00-10:00	Zheng Xiao	Degeneracy of integral points and integral solutions in Diophantine approximation
10:00-10:20	Coffee Break	
10:20-11:20	Chunhui Liu	Slope method in Arakelov geometry I
11:30-12:30	Chunhui Liu	Slope method in Arakelov geometry II
12:30-13:30	Lunch Break	
13:30-15:30	Free discussion	
May 11 <sup>th</sup> , 2023 (Thursday)		
9:00-10:00	Hui Gao	On integral p-adic Hodge theory
10:00-10:30	Coffee Break	
10:30-11:30	Peng Zheng	Local positivity of line bundles
11:30-13:30	Lunch Break	
13:30-14:30	Chunhui Liu	Slope method in Arakelov geometry III
14:40-15:40	Chunhui Liu	Slope method in Arakelov geometry IV
15:40-16:00	Coffee Break	
May 12 <sup>th</sup> , 2023 (Friday)		
9:00-10:00	Runlin Zhang	Counting integral points on homogeneous varieties and homogeneous dynamics
10:00-10:20	Coffee Break	
10:20-11:20	Yao Li	Categorification of Harder-Narasimhan Theory
11:30-12:30	She Yang	Dynamical Mordell–Lang conjecture for totally inseparable liftings of Frobenius
12:30-13:30	Lunch Break	
13:30-15:30	Free discussion	

# Titles and Abstracts

**Shilei Fan (Central China Normal University)**

*Bohr chaoticity of dynamical systems*

In this talk, I will introduce the notion of Bohr chaoticity, which is a topological invariant, and is opposite to the property required by Sarnak's conjecture. Such a system is by definition never orthogonal to any non-trivial weight and it must be of positive entropy. But having positive entropy is not sufficient to ensure the Bohr chaoticity. We proved the Bohr chaoticity for all topological dynamical systems which have Horse-shoes, all toral affine dynamical systems of positive entropy. However, uniquely ergodic dynamical systems are not Bohr chaotic and there are many such dynamical systems of positive entropy. At the end, I will discuss the complexity rational dynamical systems on  $p$ -adic fields.

**Hui Gao (Southern University of Science and Technology)**

*On integral  $p$ -adic Hodge theory*

In complex geometry, one uses Hodge structures to encode the linear algebraic structures of singular and de Rham cohomologies. In this talk, we construct a category of Breuil-Kisin  $G_K$ -modules to encode the (semi-) linear algebraic structures of the integral  $p$ -adic cohomologies recently developed by Bhatt--Morrow--Scholze and Bhatt--Scholze. These modules also classify integral semi-stable Galois representations.

**Weikun He (Academy of Mathematics and Systems Science, CAS)**

*Spectral gap in some finite groups*

Uniform spectral gap (of the discrete Laplace operators) of a family of finite groups is also known as expansion in groups or the property ( $\tau$ ). In the case of  $SL_2(\mathbb{Z}/n\mathbb{Z})$ , a uniform spectral gap is present as a consequence of Selberg's  $3/16$  theorem and the Burger-Brooks correspondence. In this talk, I will explain how to show uniform spectral gap in broader classes of groups, using combinatorics and dynamics. This talk is based on a joint work with Nicolas de Saxce, which is build on works of Bourgain-Gamburd and Bourgain-Varju among others.

**Zhuchao Ji (Westlake University)**

*Local rigidity of Julia sets I & II & III*

This is a joint work with Junyi Xie. We find criteria ensuring that a local (holomorphic, real analytic,  $C^1$ ) homeomorphism between the Julia sets of two given rational functions comes from an algebraic correspondence. For example, we show that if there is a local  $C^1$ -symmetry between the maximal entropy measures of two rational functions, then probably up to a complex conjugation, the two rational functions are dynamically related by an algebraic correspondence. The holomorphic case of our criterion will play an important role in the authors' recent proof of the Dynamical André-Oort conjecture for curves.



**Yao Li (Peking University)**

*Categorification of Harder-Narasimhan Theory*

The notion of Harder-Narasimhan filtration was firstly introduced by Harder and Narasimhan in the setting of vector bundles on a non-singular projective curve. Curiously, analogous constructions have been discovered in other branches of mathematics which motivate categorical constructions of Harder-Narasimhan filtration. In this talk, we will introduce a categorical construction of Harder-Narasimhan filtration via slope method which does not need a degree function. With a theorem of existence and uniqueness of Harder-Narasimhan filtration in our categorical setting, we give a categorical interpretation of Stuhler-Grayson filtration in the case of non-necessarily Hermitian normed lattice.

**Chunhui Liu (Harbin Institute of Technology)**

*Slope method in Arakelov geometry I & II & III & IV*

In this mini-course, we will give a brief introduction to the slope method in Arakelov geometry developed by J.-B. Bost. We will also introduce its applications in height functions and equidistribution problems.

**Zheng Xiao (Peking University)**

*Degeneracy of integral points and integral solutions in Diophantine approximation*

It is a fundamental question to ask the distribution of integral solutions to Diophantine equations, which leads to the study of the distribution of integral points on algebraic varieties. In this talk, I will give a brief introduction of the problem formulated under height machinery. We will then discuss two essential tools in Diophantine approximation: the celebrated Subspace Theorem and the filtration method, with various improved versions. Recent results on the approximations and degeneracy of integral points will be outlined as well. In the end, I will introduce the most recent result of keeping good approximations while losing the restriction of being general position. Joint work with Huang and Levin.

**Junyi Xie (Peking University)**

*DAO for curves I & II & III*

The lecture is based on the recent work of Ji and Xie. We prove the Dynamical André-Oort (DAO) conjecture proposed by Baker and DeMarco for families of rational maps parameterized by an algebraic curve.

**She Yang (Peking University)**

*Dynamical Mordell–Lang conjecture for totally inseparable liftings of Frobenius*

We prove that if  $K$  is a complete algebraically closed non-archimedean valuation field of positive characteristic and  $f$  is an endomorphism of the projective space over  $K$  which is totally inseparable

and behaves as the Frobenius on the special fiber, then  $f$  satisfies the dynamical Mordell–Lang (DML) property.

**Hexi Ye (Zhejiang University)**

*A uniform bound for the number of common preperiodic points*

Let  $f_c(z) = z^2 + c$  be a family of quadratic polynomials parameterized by complex numbers  $c$ . It was known that for any two distinct parameters  $c_1$  and  $c_2$  over the complex numbers, there are only finitely many preperiodic points of  $f_{c_1}$  and  $f_{c_2}$  in common. In this talk, we are going to show that there is a uniform bound for the number of common preperiodic points as  $c_1$  and  $c_2$  vary.

**Shucheng Yu (University of Science and Technology of China)**

*The light cone Siegel transform, its moment formulas and applications*

The classical Siegel transform is a transform which takes functions on the Euclidean space to functions on the space of lattices. In this talk I will discuss a new type of Siegel transform where the role of the Euclidean space is replaced by the light cone of a certain indefinite integral quadratic form. In this setting one can use the spectral theory of incomplete Eisenstein series to prove explicit first and second moment formulas for this transform, generalizing the classical results of Siegel and Rogers. I'll then discuss some applications of this moment formula to Diophantine approximation on certain quadratic surfaces, including spheres. This is joint work with Dubi Kelmer.

**Runlin Zhang (Chongqing University)**

*Counting integral points on homogeneous varieties and homogeneous dynamics*

Consider a variety  $X$  over rational numbers equipped with a transitive action of a linear algebraic group, identified with  $G/H$ . Assuming that it has at least one integral point, one would like to have an asymptotic count on integral points. It has been observed that the measure rigidity of unipotent flows can help answer this question when  $H$  is large. We will review this connection and explain what still remains unknown.

**Peng Zheng (Université Paris Cité)**

*Local positivity of line bundles*

The base locus of a linear series is a fundamental notion in birational geometry. The augmented base locus is refinement of the base locus which capture local positivity of line bundle and linear series. It was studied by Ein-Lazarsfeld-Mustață-Nakamaye-Popa, Boucksom-Cacciola-Lopez, Birkar and others. In this talk, we will discuss the local positivity of graded linear series, adelic metrized line bundle and height functions and apply it in the distribution of small points.