Conference Schedule

	Monday	**Tuesday**	**Wednesday**	**Thursday**
9:30-10:30	Laurent Manivel	Jun-Muk Hwang	Ivan Arzhantsev	Dimitry Timashev
10:30-11:00	Tea Break			
11:00-12:00	Wahei Hara	Changzheng Li	Jarosław Buczyński	Duo Li
12:00-13:30	Lunch Break			
13:30-14:30	Shigeru Mukai		Luis Solá Conde	Sergey Gaifullin
14:30-15:00	Tea Break			
15:00-16:00	Xun Yu		Pedro Montero	Alexander Perepechko
16:15-17:15	Jaehyun Hong		Akihiro Kanemitsu	Yulia Zaitseva

Titles and AbstractsTitles and Abstracts

Ivan Arzhantsev (HSE Univ)

Flexibility, ellipticity after Mikhail Gromov, and images of affine spaces

We define flexible affine varieties and describe their basic properties. Using flexibility, we show that every non-degenerate toric variety, every homogeneous space of a semisimple group, and every variety covered by affine spaces admits a surjective morphism from an affine space.
Applying the ellipticity property introduced by Mikhail Gromov in 1989, we prove that a complete algebraic variety X is an image of an affine space if and only if X is unirational. This result is obtained in a joint work with Shulim Kaliman and Mikhail Zaidenberg. Further, we show that an affine cone Y admits a surjective morphism from an affine space if and only if Y is unirational, while this equivalence does not hold for an arbitrary affine variety. The talk is prepared within the framework of the project “International academic cooperation” HSE University.

Jarosław Buczyński (Polish Academy of Sciences)

Secant varieties of toric varieties and multigraded Hilbert scheme

For a smooth toric projective variety $X \subset P^N$ (in a torus equivariant embedding) and its Cox ring S, consider the secant varieties of X. The r-th secant variety is the closure of the union of all P^{r-1}'s spanned by points of X. We present an algebraic criterion for a point to be in the r-th secant variety in terms of an action of S called "apolarity action". The criterion heavily involves the geometry of multigraded Hilbert scheme of S, and provides a nice and uniform description of all points of the secant variety. Moreover, it is consistent with the action of the automorphism group $Aut(X)$ on the secant variety and on the multigraded Hilbert scheme. It provides a new technique for determining if a given point is in some specific secant variety, which is especially efficient if the point is invariant under a relatively large subgroup of $Aut(X)$. As an application, we are able to determine the minimal r such that a prescribed point is in the r-th secant variety, in a bunch of situations of interest to complexity theory.
The talk is based on a joint work with Weronika Buczyńska,
https://doi.org/10.1215/00127094-2021-0048 (Duke Mathematical Journal, 2021).

Sergey Gaifullin (HSE Univ)

Maximal tori of various dimension in the automorphism group of an affine variety

The talk is based on a joint work with M. Petrov. Let X and Y be two affine algebraic varieties. The generalized Zariski cancellation problem asks, if isomorphism of the products of these varieties by a line implies that these varieties themselves are isomorphic. There are some counter-examples to this conjecture. The first of these counter-examples was built by Danilewski in 1989. In the same work due to Danilewski, it was shown that the presence of counter-examples to this problem often leads to the appearance of non-adjoint maximal tori in the automorphism group of the cylinder, that is, the product of the variety X on a line.
In the talk an algebraic technique that allows us to build new counter-examples to generalized Zariski cancellation problem will be described, as well as a technique for proving the non-conjugacy of tori. In particular, we show how to construct an affine variety with maximal tori of various dimensions in the automorphism group. As far as the authors know, such an example was not known before. The talk is prepared within the framework of the project ‘‘International academic cooperation’’ HSE University.

Wahei Hara (IPMU)

Rank two weak Fano bundles over Fano threefolds of Picard rank one

This talk discusses the classification of rank two weak Fano bundles over arbitrary Fano threefolds of Picard rank one. The result gives the generalisation of the result by Muñoz, Occhetta, and Solá Conde on the classification of rank two Fano bundles over those Fano threefolds. The contents of this talk depends on three joint works with T. Fukuoka and D. Ishikawa.

Jaehyun Hong (IBS)

Geometry of Lusztig varieties

In a series of papers, Lusztig developed a theory of characters of a reductive algebraic group G by using perverse sheaves on G. To get appropriate perverse sheaves on G (called character sheaves), he considered a family of subvarieties of the flag variety G/B parameterized by elements in G; now, we call Lusztig varieties. In this talk, we will explain how they are related to two interesting families of subvarieties of the flag variety, Schubert varieties and Hessenberg varieties. Lusztig varieties share many nice properties with Schubert varieties. They are normal Cohen-Macaulay and have rational singularities, and nef line bundles have vanishing cohomology in positive degrees. Also, we construct a flat degeneration of Lusztig varieties to Hessenberg varieties and compare their cohomology spaces. The cohomology space of a Hessenberg variety admits an action of the Weyl group, which gives combinatorial invariants of the

corresponding unit interval graphs and character values of the Hecke algebra when G is of type A. We expect our results will provide a way to extend these relations to the case when G is of arbitrary type. This is joint work with P. Brosnan and D. Lee.

Jun-Muk Hwang (IBS)

Minimal rational curves on equivariant group compactifications

Let X be a nonsingular equivariant compactification (as a symmetric space) of a simple algebraic group G. We show that minimal rational curves on X are orbit-closures of 1-parameter subgroups of G and the set of minimal rational curves through a general point is the closure of an adjoint orbit. This generalizes a result of Brion and Fu's on wonderful group compactifications to arbitrary equivariant group compactifications. This is a joint work with Qifeng Li.

Akihiro Kanemitsu (Tokyo Metropolitan Univ)

Quintic del Pezzo threefolds in positive or mixed characteristic

We will show that, over an arbitrary base scheme, quintic del Pezzo threefolds V5 are classified by non-degenerate ternary symmetric bilinear forms. As applications, we will discuss the geometry of quintic del Pezzo threefolds in characteristic $p > 0$, especially when $p = 2$. If time permits, we will discuss other applications, including finiteness results of V5 over number fields/rings of integers. (Based on joint work with Tetsushi Ito, Teppei Takamatsu, Yuuji Tanaka.)

Changzheng Li (Sun Yat-sen Univ)

On the semisimplicity of quantum cohomology

In this talk, we will discuss the semisimplity of the (small) quantum cohomology ring of a Fano manifold X, and will provide a simple necessary condition in terms of Betti numbers. We will also propose a conjectural quantum hard Lefschetz for a hypersurface Y inside X of Picard number one, saying that the induced algebra morphism on the classical cohomology can be lifted to the quantum cohomology. We then completely determine the semisimplicity of the quantum cohomology of a smooth hyperplane section of a complex Grassmannian, where we will see the ADE classification as well as evidences for our conjecture. This is based on my joint work with Sergey Galkin, Naichung Conan Leung and Rui Xiong.

Duo Li (Sun Yat-sen Univ)

Uniform bundles on rational homgeneous varieties of Picard number 1

Let X be a rational homogeneous variety of Picard number 1. We consider uniform bundles on X. We obtain an upper bound $a(X)$ such that if the rank of a uniform bundle E is smaller than $a(X)$, then E is a direct sum of line bundles. For most cases, we show that the upper bound $a(X)$ is optimal and we classify unsplit uniform bundles of minimal rank.

Laurent Manivel (Univ Paul Sabatier)

Some birational involutions of Hilbert schemes of points on K3 surfaces

The existence of regular or birational automorphisms of Hilbert schemes of points on K3 surfaces can be decided by lattice theoretic considerations, but this gives no insight on their geometry. I will present a simple construction of birational involutions generalizing the classical Beauville involutions, and study their properties. The case of the Hilbert scheme of three points on a K3 surface of genus 10 is best described in terms of the geometry of the adjoint variety of the exceptional Lie group G_2. This is a joint work with Pietro Beri.

Pedro Montero (Univ Técnica Federico Santa María)

Automorphisms and Hodge structures of Klein hypersurfaces

The Hodge structure of a smooth projective algebraic variety is an important invariant which, in many cases, is expected to determine the variety itself. This expectation is called the "Torelli principle", and it is based on a classical theorem by Torelli for algebraic curves and their Jacobians. In this talk, we will focus on Klein hypersurfaces in the projective space and we will explain how to determine their automorphism group and, in some cases, how to compute the automorphism group of the associated polarized Hodge structure. These results provide new positive evidence for the Torelli principle for cubic fivefolds and quartic threefolds, for which we can associate a principally polarized abelian variety called the Intermediate Jacobian. This is based on a joint work with Víctor González (UTFSM), Álvaro Liendo (U. de Talca) and Roberto Villaflor (UTFSM).

Shigeru Mukai (RIMS & MCM)

K3 surfaces, graphs and group actions

As complex analytic surface, K3 is a deformation of a quartic surface in the projective 3-space. Special ones are characterized by a configuration of smooth rational curves on them and also by a finite group action (preserving a polarization of certain degree). I will discuss several sample cases and three generalizations:
(i) configuration of irreducible (-2)-divisors on higher dimensional holomorphic symplectic manifolds of type $K3^{[n]}$ (and OG10),
(ii) P^1-configuration on K3 surfaces over a field of positive characteristic $p > 0$, especially on supersingular ones, and
(iii) (-2)-configurations on symplectic manifolds in positive characteristic $p > 0$.
In the last case, two strongly regular graphs of Higman-Sims and McLaughlin appear and two sporadic finite simple groups HS and McL are expected to have a birational action in the last case.

Alexander Perepechko (HSE Univ)

Automorphisms of affine varieties: flexibility and unipotent group actions

A G_a-action on an affine variety X is an algebraic action of the additive group G_a (K) of the base field K. A variety is called flexible, if for each smooth point, its tangent space is spanned by tangent vectors to orbits of G_a-actions. We will discuss the connection of flexibility and multiple transitivity of the automorphism group action. We will also survey families of varieties known to be flexible.
In the case of an affine space A^n, there is a natural notion of a subgroup of triangular automorphisms, which is an infinite-dimensional analogue of upper-triangular matrices U(n) in the matrix group GL(n). It is well known that any unipotent subgroup of GL(n) is conjugated to a subgroup of U(n). Unfortunately, this result does not hold for the subgroup of triangular automorphisms.
We will present a generalization of a triangular automorphism subgroup for an arbitrary affine variety X that describes all maximal unipotent subgroups of Aut(X). We will also discuss its properties, construction, and connection to additive actions. In particular, any unipotent subgroup of Aut(X) happens to be closed in the Zariski topology. The talk is prepared within the framework of the project "International academic cooperation" HSE University.

Luis Solá Conde (Univ Trento)

Birational maps associated to torus actions

Given an action of the 1-dimensional complex torus on a normal projective variety, one gets a set of birationally equivalent GIT quotients. In this talk we will consider the opposite question: given a birational map, we want to realize it as the induced birational map between two GIT quotients of a projective variety by a torus action. We will present examples of this two-way correspondence and discuss its combinatorial counterpart, appearing in the context of toric varieties.

Dimitry Timashev (Moscow State Univ)

On the orbit space of a maximal compact subgroup acting on a spherical homogeneous variety

Let $X = G/H$ be a spherical homogeneous variety for a complex reductive algebraic group G. We consider the action of a maximal compact subgroup $K < G$ on X. Victor Batyrev conjectured around 2016 that the orbit space X/K is homeomorphic to the so-called valuation cone V of X, which is a convex polyhedral cone whose rational points parametrize G-invariant discrete *Q*-valued valuations of the field *C*(X). (This conjecture was announced only recently in arXiv:2403.09091.) The "non-archimedian" version of this conjecture holds true, due to D. Luna and Th. Vust (1983): the set V(*Q*) is in bijection with $X(F)/G(O)\times Aut(O)$, where F is the field of Puiseux series (= the algebraic closure of *C*((t))) and O is its ring of integers. We prove Batyrev's conjecture and discuss relation between the face stratification of V and the orbit type stratification of X/K.

Xun Yu (Tianjin Univ)

On automorphism groups of smooth hypersurfaces

We show that smooth hypersurfaces in complex projective spaces with automorphism groups of maximum size are isomorphic to Fermat hypersurfaces, with a few (explicitly given) exceptions. This is a joint work with Song Yang and Zigang Zhu.

Yulia Zaitseva (HSE Univ)

Additive action on projective hypersurfaces

We will provide an overview of the results on open embeddings of affine space A^n into complete algebraic varieties X, for which the action of the vector group G on A^n by parallel translations extends to an action of G on X. We will begin with the Hassett-Tschinkel correspondence, which describes equivariant embeddings of A^n into projective spaces, and we will present its generalization to the case of embeddings into

projective hypersurfaces. It has been proven that non-degenerate projective hypersurfaces that allow such embeddings correspond to Gorenstein local algebras. Moreover, such an embedding into a projective hypersurface is unique if and only if the hypersurface is non-degenerate. Furthermore, we will mention results concerning embeddings into complete toric varieties, into flag varieties and their degenerations, and into Fano varieties of certain types. The talk is prepared within the framework of the project “International academic cooperation” HSE University.